# How To Sort

## A short guide on sorting investigations

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# Contents

## 1 Introduction

Classification is essential for human life. It enables the individual to orient himself. Subliminal as well as conscious processes permanently classify stimuli and sort out stimuli of interest, initiating and controlling behaviour. A well-sorted environment facilitates orientation, therefore classification matters to design.

Sorting is a natural way to classify, and it is an everyday activity. This presentation aims at presenting basic knowledge, or at least sources of knowledge about sorting as a method of research and design. Since a monograph on sorting already exists (Coxon, 1999), this paper does not fully cover and discuss the subject in depth. The intention is to show the basic ideas behind the method, the resources available and what actions need to be taken for research.[1] The chapters follow the natural steps of research, from a decision to make use of the sorting method through to data analysis.

Sorting results can be represented in a number of ways, such as cluster lists, lists of group members, dendrograms or general graphs, Venn diagrams and other forms (Coxon, 1999). Mathematically speaking, these different representations are at least partially equivalent. Some of these representations such as trees and Venn diagrams can be used for data collection as well as for data analysis. Analysis of sorting results most often means no more than transforming data from one representation to another. From this point of view, sorting is just another application of graph theory, set theory and combinatorics.

## 2 Reasons for doing a sorting investigation

Sorting is a convenient way to evaluate how people organise a set (or “domain”) of well- defined entities (items), which may be perceptions or objects or concepts. This information may be obtained for design, market research, sociological or psychological research purposes. Sorting is a comparatively simple task for both participants and researchers, with most variants requiring only judgments by the participants about assignment of items to groups, categories or attributes. It does not require any grading, ranking or scaled rating and therefore the issue of establishing a psychologically valid and reliable scale does not arise. Since it does not require any paired comparisons, sorting lends itself to a relatively large number of items, with examples of up to 400 items known from literature. According to literature a stable averaged model of a domains structure is usually obtained with some 20 to 30 participants of an investigation.

Models of human cognition and memory supported by the sorting method include partitions, fuzzy sets, cover sets, trees and networks. A number of statistical analysis methods and associated freeware or shareware software is at hand to analyse two-way and three-way data obtained using these models and to produce some data representation using these models.

The cognitive structure is assumed to remain stable at least for the time period from the beginning of the evaluation to the implementation of results (e.g., of a web site by visitors whose model of the site had been investigated).[2]

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[1] Rugg and McGeorge (1997, reprinted 2005) published an useful introduction with special emphasis on repeated (multiple) sorting and Personal Construct Theory.

[2] The methods of sorting mentioned here are not designed for data which are systematically affected by patterns or by Gestalt Psychology (where it is assumed that the whole is more than the sum of the

# 3 Preparing a sorting investigation

## 3.1 Target determination

The first step of an investigation will be to define its target. The target in turn will largely determine which variant of sorting and which analysis technique will be employed. Typical targets of sorting investigations include the following:

- The simplest task of sorting is that of assigning items to pre-established categories (closed sorting[3]). It is much like rearranging a mixed assortment of screws. Applications of closed sorting abound in everyday life and design, e.g. placement of goods in shops or of documents in file systems or of persons in tax classes, indexing of objects (books), closed questions in questionnaires etc. Closed sorting clearly results in an exclusive and exhaustive division of the objects – a nominal scale. For large sets of items, a sorting questionnaire will be a convenient variant of the sorting method, and histograms of counts of assignments of each item may suffice for analysis.

- Sometimes it may also be desirable to identify prototypes or exemplars as best representatives of the categories or groups. In web-design, identifying them will help in establishing logos and alerts as well as featuring as landmarks in a usable navigation. Biological (Linnean) taxonomy for example relies on prototypical specimen, species and genus. - Different people uniformly identify prototypes in certain groups and most often identify genuine prototypes as prototypes. Prototypes share all or most characteristics of a group. Counting may identify them, but some Cluster Analysis techniques (e.g. ‚pam' by Kaufman and Rousseeuw, 2005) also allow for identification of group "medoids". Alternatively, respondents may be asked to identify (after completing their sorting) which item or items are „best examples of" each (non-singleton) group.

- Another task of an investigation may be to find some optimal or average representation of the structure of certain items of a domain. The 'Items' and 'domains' may for example be persons or roles within an organisation (sociology), words of a semantic domain (linguistics), chapters or pages of a text or web site as well as goods of an (internet) shop (information architecture), species of a zoological group (taxonomy) and so on.

- If, in contrast to closed sorting, the categories are not known in advance, open (or free-) sorting is used. The target is now to identify the categories as well as the final assignment of items to them. If groups are included in super groups, maybe in several hierarchical levels, variants of sorting producing an hierarchical tree are used (employing an ordinal scale of levels). If there is only one level of groups, variants producing a partition, a fuzzy set or a single-level cover set are used.

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parts). If a pattern or a gestalt influences a sorting, the result will not be interpretable through mathematical analysis but only through qualitative interviewing and understanding or through experiments.

[3] One of the authors (Harloff) prefers the term closed sorting here, which has been used in this sense in HCI research and information architecture. But fixed sorting has been used as well, which is restricted here to fixed *numbers* of categories.

For data analysis, it is important if the occurrences of items are restricted to one group or category. Such data will be analysed by the most common Cluster Analysis methods for hierarchies and partitions. If items can be placed in more than one group simultaneously, sorting results in a structure called covering set (or cover set for short) with overlapping clusters. Such data will be analysed using Network Analysis, Cover Set Cluster Analysis or Fuzzy Cluster Analysis. For Fuzzy Cluster Analysis, each item is divided between groups, while for Network Analysis and Cover Set Cluster Analysis each item is a full member of each group. (Cover Set Cluster Analysis as a term is used here for any Cluster Analysis method with cover sets as an output.)

- Other sorting investigations will aim at gathering information about the facets or criteria which people use for categorising or evaluating items. This is a common situation in market research. There may be an interest in identifying characteristic values or anchors on the facet dimensions as well, resulting in a matrix of item categories (employing several nominal scales; but anchors can also be placed on an interval scale). This approach is of interest in information architecture, if a presentation scheme for dynamic contents has to be developed. While a number of sorting variants may be employed, multidimensional scaling or some combination of Q-Mode and R-Mode scaling techniques (and two-way two-mode joint analysis) will be appropriate analysis tools (CATSCALE, DeSarbo, Libby and Jedidi, 1994). The results of scaling techniques may also be used as input for a discriminant analysis connecting everyday language concepts to abstract dimensions.

- For some purposes it is interesting to investigate the similarities and differences between sorting results from different persons (and/ or facets; Q-Mode analysis). First, it may become necessary to control the quality of the data and identify outliers. Second, it may become necessary to identify subgroups within the target population and adjust goods or offers to those subgroups (market research, information architecture). For these purposes the “minimum move” dissimilarity measures of Arabie and Boorman (1973) may be used in connection with Multidimensional Scaling.

- Sometimes, it may be desirable to look at the interactions of participant, facet and item structure in order to understand people’s models of a domain. This requires more sophisticated analysis techniques as discussed by Coxon (1999). A simple approach could repeat the analysis for each participant, facet and item combination that is a priori of interest.

A qualitative distinction can also be made in terms of the audience of the study results. If the audience consists of a narrow group of experts such as researchers, results such as complex dendrograms or scaling on multiple dimensions may be presented. If, on the other hand, sorting results are used for the design of instruments, taxonomies, or structures, that design must suit the target population such as the general public. Often, this will require a simple structure of common language terms or actions. It may be developed through simple methods such as partition sorting, or simplified from complex results using qualitative reasoning.

## 3.2 Model selection

The target chosen is intimately related to the choice of the appropriate mathematical model and therefore to which sorting variant is best suited. On the other hand, it may be not immediately clear if all members of a target population share the same way or scale of structuring a domain between each other and with the researcher. For example, everyday language may categorize colours using a set of colour names (a partition or nominal scale) possibly respecting mixtures of colours (a fuzzy set), while natural scientists may use a spectrum of wavelengths and intensities (ratio scales) to characterise light. Intuitively, a large part of participants in a study of Harloff (2005a) used an interval scale to characterise colour similarities. (Figure 1 gives another simple example of different valid ways to structure a domain.) Therefore, it may be necessary to determine a model appropriate for the purpose of investigation but also for the target population as a first step of investigation. A short preliminary qualitative interview on the properties of the domain may be helpful (e.g. concerning multiple placements or ties of items, presence or absence of hierarchical levels) but also a comparison of goodness of fit measures of statistical results using several different models.

Nominal data can be gathered through open or closed partition sorting, suited for relationships of equivalent items and categories without a hierarchy. Especially for networks, cover sets or fuzzy sets, items may exhibit multiple relationships or multiple assignments to categories (fuzzy sorting). If the groups of items are built based on a rank order of items according to a scale (such as age-groups of people, size-classes of screws etc.) the process is called graded sorting. As a difference to partition sorting and fuzzy sorting graded sorting necessarily implies the provision of a scale (criterion) for sorting by instruction.

Ordinal hierarchical level data, such as software menu structures, can be found through hierarchy construction and through weighted sorting. A major difference between those two is that hierarchy construction implies the assumption that all single items share the same level of abstraction, while weighted sorting can deal with items from different levels of abstraction. Except for graded sorting these methods and structures imply discontinuous scales and data, and categorisation as the basis of sorting. Interval scale structures and continuous data on hierarchical levels, on the contrary, may be investigated using weighted sorting as well (Harloff, 2005a). Whilst weighted sorting has not been extensively investigated, it is assumed to be suited for the construction of instruments for manipulation of multimedia output or machines in information architecture. For continuous data, sorting is based on judgements of relative similarity.

## 3.3 Sampling

Just as in any social science research, preparing a sorting investigation includes defining the target population, the sampling procedure and the sample size. Consequences may be inferred from the population about the kind of stimuli and media. Computer based sorting methods, for example, require a computer-literate sample of participants, which nonetheless represents the target population well. Therefore, computer based methods will not be suited for every population. On the other hand, Internet based sorting methods offer the opportunity to reach an international sample of participants with little effort, compared to travelling or charging international research companies.

Several authors have looked for an optimal sample size to choose. In a set of 50 illustratory examples of sorting experiments (Coxon 1999, Table 4, pp 85-87) sample sizes vary between 5 and 200, with a median of 50. A major criterion has been the stability of averaged sorting results, and a high similarity of averaged sample results to the results obtained from very large samples (representing the total of a population well). Miller (1969) recommends using a sample of 20 participants. Tullis and Wood (2004) recommend 20 to 30 participants, leading to an average correlation of 0.95 between results of subsets of this size and results of 168 participants. Nielsen (2004) regards a correlation of 0.9 as sufficient which, based on the data of Tullis and Wood (2004), requires a sample size of 15. Replications of these investigations are needed in order to evaluate the dependency between correlations, domains, sorting methods and number of items.

### 3.4 Domain and item definition

A further preparatory step is the definition of the domain investigated and of items (stimuli) making up the domain. As Coxon (1999) pointed out, a domain may be defined ("intensively") by a rule that allows for judgement about membership of items to that domain, or ("extensively") by exhaustive enumeration of members. The definition will often depend on culture and context; it may be the result of a pilot study.

Items are entities, which may be perceptions, objects or concepts. The question of what constitutes an object is considered here as a matter of perceptional psychology, gestalt psychology and human pattern recognition, while concepts and terms are taken as linguistic and social phenomena. Items may themselves constitute a domain composed of lower level items.

The traditional way of sorting is to manipulate physical objects or paper cards. Using paper limits the method to visual and some haptic stimuli. These can be figures or terms or lengthy descriptions of concepts. Objects can mean anything from specimen of animals (Boster, 1987) to more or less complex smelling or tasting or touchable or hearable (Bonebright, 1996) physical objects (in practice, it will be hardly possible to manually move perceptions without moving the body of related objects). Different sensual modes, however, are restricted to different conditions of memory. Computer based methods offer the opportunity to sort multimedia presentations such as videos (e.g., of actions) or auditory stimuli (Bonebright, 1996). Through sorting stimuli, objects or concepts can be assigned to groups, but also to attributes. Conversely, groups or attributes can be assigned to stimuli, concepts and objects as well (Coxon, 1999).

The set of items chosen should preferably exhaust the domain. At least if sorting aims at the construction of partitions or trees, items should be mutually exclusive (not necessarily for additive trees, Corter 1996). Partitions additionally imply the items to share the same level of abstraction (such as simple, single sensual stimuli). Initially, a set of items may be constructed with subjects using free listing, mind maps, concept maps, interviews, focus groups etc. Usually, the item set must be overworked in order to meet the cited requirements. Verbal presentation of items should be concise, unequivocal and simple. It may become necessary to test the understanding of verbal items prior to sorting. It may also be necessary to equate items for which synonyms exist, as for instance where there is a variety of "street-terms" which refer to the same recreational drug. If, alternatively, sorting is based

on the similarity of items on continuous scales, the set of items should be diverse with respect to any important facet, covering a wide range of values.
Consequences may be drawn on the sorting variant best suited for the domain. The task of sorting several hundreds (or even thousands) of items freely within only one investigation may, for example, be too taxing to participants. With vast numbers of items, therefore, it may be more appropriate to define categories from results of a free sorting pilot investigation, followed by another investigation with fixed categories using a sorting questionnaire including all items. Computer based variants of free sorting will become cumbersome with large sets of items whose representations and/ or groups do not fit on one computer screen. The necessity to scroll may lead to a taxing overload of memory and therefore to distortion of the sorting results. On the other hand may questionnaire sorting be faster and easier to accomplish for closed sorting and large item sets than card manipulation.

### 3.5 Instruction and observation preparation; test runs

Materials for presenting the items to participants must be prepared in advance. Depending on the variant chosen such materials may be paper cards, paper questionnaires, objects, special computer files required by sorting programs, or HTML files, script files and databases, as well as data analysis tools. If paper cards are used and items may be placed in several categories simultaneously, several equal sets of paper cards or a fast replication device may be necessary. For drawing of contour lines around groups, large sheets of paper, pencils and a large board may become necessary. A camera may be appropriate for storage of paper card sorting results, but also envelopes, rubber bands, clips and the like.

It will normally be necessary to produce guidelines for observers and interviewers as well as instructions and questionnaires for participants. Observations and interviews may be required in order to understand what sorting results mean and how they were produced as well as to obtain appropriate item, category, level and facet labels and instances of prototypical elements of categories that subjects form. Observations may also provide data about the sorting strategy and invested time, cognitive effort and difficulties, rearrangements and corrections of participants. While qualitative questions of an interview can be presented through electronic questionnaires, observational data cannot be gained as easily and reliably using computer-based methods.
The appropriateness of the chosen sorting variant, instructions, participants, categories and items may be tested through test runs. Furthermore, results of one sorting run may serve as input to another later one if, for example, the preparation of the latter requires a priori knowledge about categories and/ or facets.

# 4 Conducting a sorting investigation

## 4.1 Variants of sorting procedures

### *4.1.1 General*

A large number of variants of the sorting method exist (see Coxon, 1999), and only a selection is presented herein. Sorting methods can be categorised according to the cognitive and mathematical model used for division of a domain (a tree, a partition, a fuzzy set, a cover set or network) and according to the materials and actions necessary for sorting. The structure of this chapter follows this categorisation. Table 1 provides a scheme for classification of sorting investigations including the most common variants of sorting. Figure 1 applies different models to a single item set.

*Table 1) Scheme for classification of sorting investigations and choice of an appropriate variant according to mathematical model and limitations imposed by instructions*

<table>
<tr><td colspan="2" rowspan="3"></td><td colspan="4">Unique assignment of items to groups</td><td colspan="2">Option to assign items to several groups (Fuzzy Sorting)</td></tr>
<tr><td rowspan="2">Rank order of groups (Graded Sorting)</td><td rowspan="2">One level of groups (Partition Sorting)</td><td colspan="2">Hierarchical levels of groups (tree model)</td><td rowspan="2">Some items lie intermediate between groups -> several partial group memberships of these items (fuzzy sets)</td><td rowspan="2">Groups overlap (cover set) and / or items bear several relationships (network) -> several full group memberships of items</td></tr>
<tr><td>All items share the same hierarchical level. Levels are rank ordered (Hierarchy Construction)</td><td>Items need not share the same hierarchical level. Levels are rank ordered or interval scaled (Weighted Sorting)</td></tr>
<tr><td colspan="2">Meaning (and therefore also number) of categories specified by the researcher (Closed Sorting)</td><td></td><td>Originally, CS was restricted to partitions</td><td></td><td></td><td></td><td></td></tr>
<tr><td rowspan="2">Meaning of categories established by the participant (Open Sorting)</td><td>Number of categories specified by the researcher (Fixed Sorting)</td><td></td><td>Originally, Fixed Sorting was restricted to partitions</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Number of categories established by the participant (Free Sorting)</td><td></td><td>Originally, Free Sorting was restricted to partitions</td><td></td><td></td><td></td><td></td></tr>
</table>

**Fig. 1a) Partition , "partition sorting"** (*colour* )

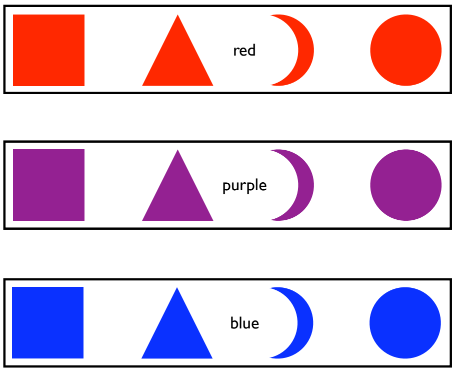


**Fig. 1b) Partition, "partition sorting"** (*shape*)

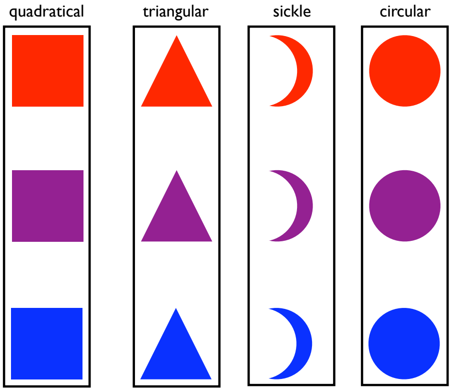


**Fig. 1c) Fuzzy sets, "fuzzy sorting"** (*shape,colour* )

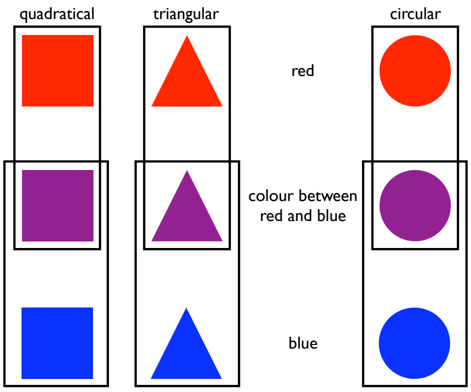


**Fig. 1d) Network, "fuzzy sorting"** (*shape,colour* )

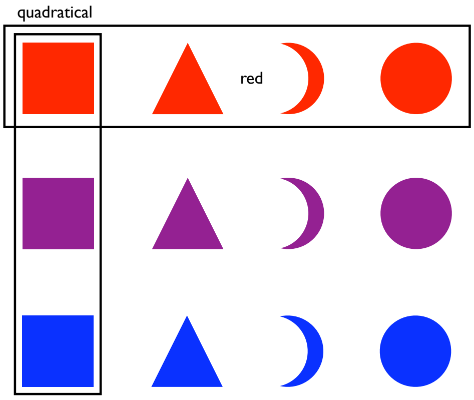


**Fig. 1e) Cover set, "fuzzy sorting"** (*shape*)

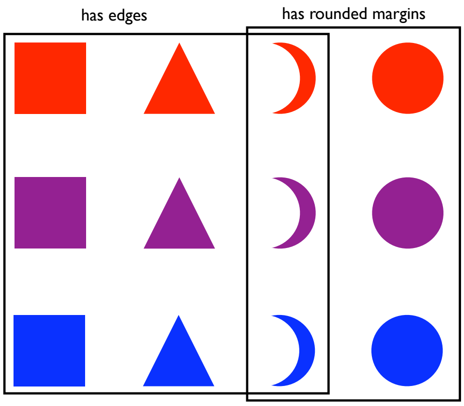


**Fig. 1f) Rank order of clusters, "graded sorting"**
(*Number of edges*)

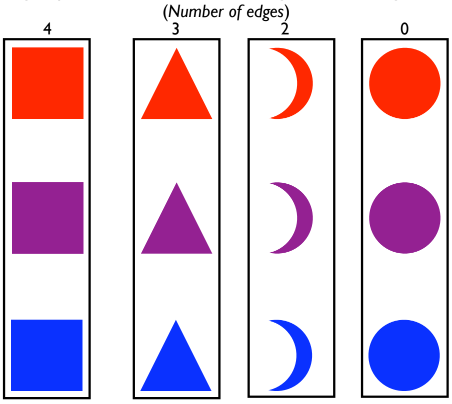


**Fig. 1g) Hierarchy, "hierarchy construction"** (*shape* )

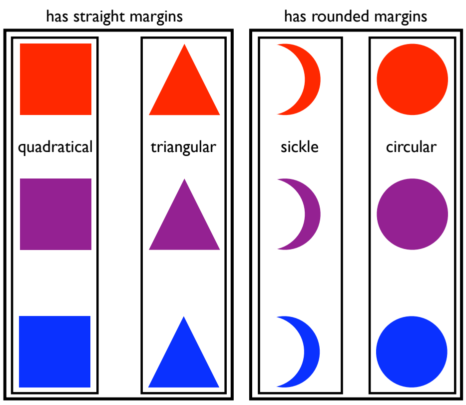


**Fig. 1h) Hierarchical fuzzy set, "hierarchy construction"**
(*shape,colour* )

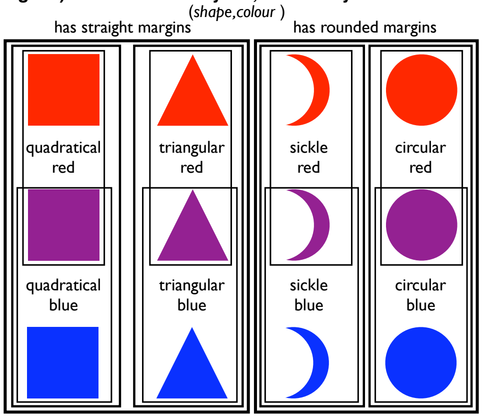

*Fig. 1 (previous page). A sample set of 12 coloured shapes sorted into clusters defined by different attributes of two facets (shape, colour) and five sorting variants from table 1. 1h) shows one of many possible combinations of sorting variants.*

Other variants allow for isolation and omission of items. Particularly during pilot studies, omissions may be a valuable source of information about the validity of item and domain definitions. Later on, statistical analysis may require full data sets. Multidimensional Scaling allows for omission, however.
Q-sorting is a sorting method not included here. It implies a specific psychological theory (Stephenson, 1953). Schmolck (1999) provided an online version for Q-sorting.

#### 4.1.2 Variants producing a partition

Closed sorting means that there are a number of pre-specified, labelled categories to sort into. Open sorting on the contrary means that the researcher does not specify categories. They are left to the subject, and for this reason the method has been called „own categories“ by Sherif, M. and Sherif, C.W. (1967). Fixed sorting specifies the number of categories, but not their labels or meanings (but see footnote 2 on page 5). Free sorting is the most common variant, leaving the meaning, number and size of categories to the participants.
Often the criteria (facets) used for assignment of cards to groups are also left to the participants, or a very general facet such as overall similarity is chosen. This approach will lead to a two-way data table and two-way analysis. It may be a target of such an investigation to find out which criteria are used through observation, qualitative questions and Multidimensional Scaling.
If, however, a faceted approach is used, it will be necessary to instruct the participants on the facets they should use for assignment. Repeated sorting for each of these facets is called multiple sorting (Rosenberg and Kim, 1975). Canter, Brown and Groat (1985), however, left the kind and number of facets to the participants. Rugg and McGeorge (1997) use a very similar approach but with repetitions according to constructs (not facets) based on Personal Construct Theory. Multiple Sorting will lead to a three-way or four-way set of data and analysis; collapsed tables will no longer represent partitions.

#### 4.1.3 Variants producing a fuzzy set or a network

For partitions, every item is placed in one, and only one, group. Unique placement of items is, however, not always appropriate for the domain. Human memory, for example, may be modelled as a network of contents connected through multiple ties. The distinctive property of Hypertext are contents being linked in multiple ways in a network. Humans are commonly members of more than one group of persons. Such models mean that each item is fully placed in several groups, mathematically modelled as networks (graphs) or cover sets. Other models mean that each item may be placed intermediate between groups of items, such as mixed colours. Mathematically speaking, these are fuzzy sets. Zaus (1997) and Ragin (2000) have demonstrated the great potential fuzzy set theory offers for the social sciences.
If multiple assignments are allowed or even desired, instructions should be given accordingly. It should be explicitly stated if multiple assignments symbolise intermediate positions between groups related to a fuzzy set or if they symbolise simulta-

neous full membership related to a cover set or network. Physically, multiple copies of cards or objects may be made available for sorting through manipulation, or intersections of contour lines around groups may be allowed, or check boxes may be used instead of radio buttons in questionnaire sorting. The latter variant has been demonstrated by Harloff (2005b) to yield results differing significantly from results of a radio button questionnaire for the same item set. The possibility to assign items to several groups is often used by participants.
Capra (2005) used a variant of sorting combining multiple assignment of items to groups and hierarchical levels of groups. This seems to be the most general and free variant for sorting available. But while a web site can indeed be designed to fit such a model, in Joachim Harloff's opinion it should be carefully considered if a model allowing single items to be placed on different hierarchical levels simultaneously makes sense for a domain. It may be necessary to exclude such a condition.

#### *4.1.4 Variants producing a tree*

A group of closely related methods for producing trees is called hierarchy construction (Coxon, 1999). There are agglomerative and divisive procedures, depending on whether sorting starts with isolated items that are sorted together in groups, or if sorting starts with a pile of all items that is divided into subgroups. Some variants force the participants to join exactly two items or groups each step, respectively divide a group into exactly two subgroups, and no two groups may be built simultaneously. The method is akin to fixed sorting on multiple levels. Whereas the results have some nice mathematical characteristics (e.g., there are always n-1 levels present with n items, therefore two trees of the same item set can be easily compared) it is moot if such a structure will be a good representation of a person's model. Other variants leave the size of the groups built to the participant, but still no two groups may share the same level of agglomeration. The variant leaving most freedom to the participant is to let participants sort items freely on the lowest level, then the groups built are freely sorted on the second level, then the super groups are freely sorted and so on until an end is reached (Coxon, 1999). For hierarchy construction, this variant is most likely to represent a participants mind well. As mentioned for fuzzy sorting, hierarchy construction can be combined with fuzzy sorting in various ways (Capra, 2005; figure 1h herein).

"Opposite sorting" was introduced by Bimler and Kirkland (2001, 2003). It involves a second step after sorting items into groups according to similarity, and the authors use it mainly in combination with variants of hierarchy construction. Participants are asked to identify pairs of groups (items), which are most *dissimilar*. This way, major axis of variance may be identified, but more importantly dissimilarity data are available for Multidimensional Scaling. According to Bimler and Kirkland (2001, 2003) a Multidimensional Scaling solution will now be more reliable, since results from this scaling technique depend largely on the greatest dissimilarity values. The greatest dissimilarities are not reliably determined if sorting results rely on similarity data from co-occurrences alone.

Agglomerative hierarchy construction implies the assumption that all items sorted share the same level of abstraction, as well as the groups built from them on higher levels. Moreover, the level assigned to a group or super group is partially predetermined by the agglomerative or divisive procedure. Harloff (2005a) introduced weighted sorting as a solution for the cases that groups of items do not all share the same level of abstraction in a participants mind. Groups are built in a free sorting manner on all levels simultaneously, while repeated rearrangements are allowed until the participant reaches a state of satisfaction with the configuration. Similar procedures restricted to two levels have been used previously by e.g. Tullis (1985) and Dong, Martin and Waldo (1999). In a second step, groups and super groups are spread over levels of internal similarity. At the end each participant has freely built his personal tree. Depending on the instruction a participant may model the levels of internal similarity using an ordinal scale (equidistant occupied levels) or an interval scale (adjusted distances between occupied levels). Weighted sorting needs replications to learn more about its characteristics for example with respect to its feasibility with large sets of items.

#### *4.1.5 Sorting by manipulation*

Sorting of paper cards has been the usual way of sorting since its advent in the 1930s. Words, phrases, photos or line-drawings are printed on paper cards. These cards are given to participants for sorting.
If objects other than stimuli or concepts are to be sorted, it may be feasible to let participants sort the objects themselves instead of representations of the objects.

Variants of card sorting producing a partition or a tree usually require cards to be sorted into stacks or piles of cards, implying clear cut group boundaries on a nominal or ordinal scale. (Without clear cut group boundaries this variant will force participants to use a threshold of similarity for assignments to groups, see for example DeSarbo, Libby and Jedidi 1994.) The stacks may be subdivided or joined subsequently as in hierarchy construction, resulting in a tree. Otherwise, there may be multiple identical cards or a facility to replicate cards, so they can be assigned to multiple stacks. This will result in a fuzzy set, a cover set or a network.

Relationships between cards may also be symbolised by the geometrical placement of cards (e.g., their distances) on a table, large board or sheet of paper. Geometrical placement in principle allows for modelling of continuously varying degrees of similarity. For hierarchical card sorting, contour lines can be drawn around groups of cards on several agglomeration levels. On the other hand, intersections of contour lines can be used to model multiple group memberships (fuzzy sorting) as long as the total figure is not too complex. It will be convenient to allow participants to evolve their concepts, repeatedly erasing and redrawing the lines. – Placement on a board does not necessarily imply nominal or ordinal scales. Weighted sorting (Harloff 2005a) extends on this idea. Groups and super groups of cards are placed along a similarity ordinate. This allows for an interval scale of measurement.

A couple of computer programs has been produced which emulate the drag and drop handling of symbolic “cards” (Bonebright, 1996; Dong, Martin, and Waldo, 1999; Edmonds, 2000; Information & Design, and UCDesign, 2001; NIST, 2004; Schilb, 2003; Wood and Wood, 2003; Wood, Wood, and Anderson, 2002; among others). Only a few of them offer the option to use more than one hierarchical level or to place cards in more than one group simultaneously. They all use the model of building piles of cards. The equivalence of results from paper card sorting versus drag and drop computer card sorting has not yet been investigated. One major dif-

ference in manipulation is that participants may move several cards simultaneously in different directions with their hands (as observed repeatedly by Harloff and others), which is not possible using one mouse and a computer screen. A large set of items will force the user to scroll on the computer screen, taxing his memory.
Additionally, a couple of computer programs for mind mapping and concept mapping exist which were not originally designed as sorting tools. They nevertheless can be used to produce trees and hierarchies as well as networks graphically, which is just another way to denote the same kinds of relationships of items (Trochim, 1989). Cooke (1994) included an account of this topic.

#### *4.1.6 Sorting by assignment*

Sorting by means of questionnaires has been described first by Coxon (1999). The core of such a questionnaire consists of a table. Items may be assigned to rows of a table, with their representations (words, phrases, figures, Hypertext links …) placed in the first column. Categories may be assigned columns of the table, with their representations placed in the first row of each page. (Of course a transposed layout can be used as well.) The first row may be repeated, at least on top of each page. The layout of the table should take care that rows and columns can be traced easily, for example through colouring. For each row participants will mark the column (or columns) the item belongs to.
It will be convenient to reserve a separate column for explicit omission of items from the domain, and a column for indicating a lack of information required for sorting. For evaluation of the results it makes a difference if a person accidentally overlooked an item or could not judge upon it, or if he explicitly states that an item does not belong to the domain or any of its categories.
For free sorting, no labels of columns are given except perhaps for numbers. Elements to be placed in the same group are marked in the same unique column. Harloff (2005b) carried out free sorting investigations using questionnaires.
Up to now, no simple way has been demonstrated for gathering information about hierarchies through questionnaires. In principle this could be done if every table cell contained a text area, and participants marked group membership by symbols (e.g., letters) unique for each group and row, producing a hierarchical clustering scheme (Johnson, 1967). This is, however, perhaps not a simple task.
Coxon (1999) has described a further questionnaire variant with only one column of text boxes next to the column of items. The rest of each row enumerates the options available for that row and text box. This variant is more flexible, since options and sets of options can be adapted to each item.

The simplicity and virtue of questionnaires shows up especially if the labels and number of categories are known in advance (closed sorting) while the number of items is large. For free sorting with large numbers of items orientation on the table and tracing of columns may become cumbersome.
Questionnaires and tables may be implemented on paper or on computer screens. Computer variants may include the use of spreadsheets or HTML forms (or scripts producing HTML code) and data bases. Radio buttons in each cell of the table will make sure that each item will be placed in just one group or category, while check boxes will be used if items can be placed in more than one group (fuzzy sorting). Scripts can be used to control and avoid errors like unintended omissions, but also for internationalisation of the questionnaire. Internationalised questionnaires will

open new resources for research on relations of cultures and cognitive concepts (like that of Boster, 1987).
According to Harloff (2005b) free sorting results from sorting questionnaires will not always be the same as those from paper card sorting. One reason for differences may possibly be different sorting strategies employed. For example can intermediate steps and corrections of assignments be more easily produced through manipulation than through multiple erasing and redrawing in questionnaires. Additionally, Harloff (2005b) observed less of a tendency to leave items isolated with paper cards than with questionnaires.

Harloff published a php script for supporting HTML forms and a MySQL database as open source code (*https://sourceforge.net/projects/formsforsorting*). The script supports multimedia content as well as internationalisation of the questionnaire.

## 4.2 Questions to ask after sorting

Asking participants some simple questions after each sorting may often help to improve the investigation following test runs as well as to understand the meaning of results. Examples of such questions include:
- Did you follow any strategy of sorting? If so, could you describe your strategy?
- Please label and describe the groups you constructed.
- Which criteria (facets) did you use for clustering? Are there more criteria, which you did not use? Which criteria are important in which context?
- What are the differences between the groups you built? Could you provide a rule for discriminating groups, or for assigning (new) items to groups?
- Is there any item (object) representing this group well (for each group)? Which represents the group best? (Why does this item represent the group best? What is so characteristic about it?)
- Were you able to express your opinions freely and correctly? If not, could you suggest any improvements for this investigation which would enable you to express yourself more freely and correctly?
- Did you feel any uncertainty concerning the procedure of sorting? (Which?)
- Did you feel any uncertainty during sorting? (When?)
- Are there any items you consider relevant to this domain that did not appear? (Which?)
- Were there any items presented which you feel do not belong to the targeted domain? (Which?)
- Are there any items that were difficult to categorise? (Which and why?)
- Do any items overlap with respect to the criteria you used for sorting? (Which? Suggested improvements?)
- Do all items share the same level of abstraction? Otherwise, some items could represent super groups of items that are equivalent to the others. (Which do not? Suggested improvements?)

# 5 Analysing sorting data

## 5.1 General

Non-hierarchical sorting methods can generate binary data on occurrence versus non-occurrence of items in groups (or, whether an item possesses a property or not; Coxon, 1982). The occurrence data can in principle be provided by participants of a study, using one of several criteria (facets) for assignment if sorting was repeated for each facet (multiple sorting). Thus, results can be written as a four-way table (items, groups, individuals, and facets) of binary data. It can be analysed in one-way, two-way, three-way and (theoretically) four-way analysis, but four-way analysis to our knowledge has never been tried. Repetitions provide a fifth dimension of data. Up to now repetitions are used for reliability considerations only, not in combination with other dimensions. However, studies of changes of concepts through time may be a future target for linguistic and social science research. Usually, a four-way data table is collapsed prior to analysis. Often the table is summed or averaged for individuals while usually no different facets are separated. (This practice may be due to the [as yet] poor dissemination of multi-way analysis techniques and software.)

For one-way analysis and closed sorting (pre-established categories), it is sufficient to count assignments of items to groups by individuals and/ or by facets to judge about the ‘correct’ placement of the items. Histograms, pie-charts and the like may be used to display and judge the relationships.

For two- and three-way analysis researchers most often transform occurrence data of items in groups into co-occurrence data of items (Coxon, 1982). Co-occurrence matrices of items in the simplest case consist of binary data (0: no co-occurrence; 1: one co-occurrence in one group) for each participant, if each item is placed in only one group and if no information about hierarchies is coded. Burton (1975) has discussed another three ways for conversion of occurrences to co-occurrence measures – by weighting by the size of the group, by weighting by the reciprocal of the size of the group, and by an information-theoretic combination of the probabilities of a pair belonging and non belonging to group. The simple use of 0s and 1s is prone to lead to degenerate MDS solutions with tightly collapsed groups of items.

Summing or averaging the respective matrices aggregates results of different individuals. Analysis then proceeds to Cluster Analysis or Multidimensional Scaling of dissimilarity matrices, but consequences arise from the fact that the data stem from binary data and partitions (see chapter 5.5 about Multidimensional Scaling for a discussion). Spectral Cluster Analysis or Principal Component Analysis of similarity data (both two-way tables and co-occurrence tables) can also be used to determine partitions.

If the sorting leads to a tree (using hierarchy construction or weighted sorting), the level (tree edge) where two items are first joined in the same (super-) group may be used as a dissimilarity measure (that is the length of the shortest path connecting them divided by two; e.g. Tullis, 1985). Alternatively, a Maximum Likelihood measure may be used (Dong, Martin and Waldo, 1999; Harloff, 2005a), but the matrix structure resulting from the two measures will usually be about the same (Harloff, 2005a). Again, the dissimilarity matrices may be analysed using Cluster Analysis and Multidimensional Scaling.

For fuzzy sorting the distance (or proximity) measures will no longer be binary either. Examples of suited similarity measures for pairs of items are the number of co-occurrences, and indices (e.g. Coxon, 1982: 27 table 2.4; Jambu and Lebeaux, 1983). The Jaccard index (Capra, 2005) and the Dice index (Harloff, 2005b) were successfully used on sorting data. Wing and Nelson (1972) presented another measure useful for traits of persons, based on arguments about psychological characteristics of traits. Dissimilarity matrices may be analysed by Cluster Analysis and Multidimensional Scaling, while similarity matrices may be analysed using Factor Analysis.

All analysis methods of co-occurrence data discussed have been usually restricted to relationships of pairs of items. They thus used only a part of the information present in sorting results. Daws (1996) presented a solution on free sorting data for triple comparisons as well as related adaptations of cluster analysis and multidimensional scaling. Cox, F., Cox, M. A. and Branco (1991) gave an account of n-tuples for Multidimensional Scaling.

## 5.2 Software

Most of the analysis methods discussed have been presented in scientific papers. The statistical analysis programs and their source code have often not been made available to the public. Therefore, if one of the more sophisticated techniques is employed, one possibly is to do some programming on ones own. Another list of available software may be found within the appendix to Coxon (1999) on the website http://www.methodofsorting.com/, which may be more complete than the information presented herein. (Readers are invited to suggest further entries.)
Tony Coxon has produced a package (SORTPAC) which takes input data in various formats, converts them into Burton measures and/ or calculates Arabie-Boorman measures of partition-distances between individual sortings (see chapter 5.7 on Q-mode Analysis). He may be contacted for help (http://www.methodofsorting.com/). ANTHROPAC precedes SORTPAC and includes free-sorting and other procedures. See http://www.analytictech.com/apacdesc.htm and Borgatti (1992, 1996) for more information about ANTHROPAC.
Joachim Harloff has set up an open source project for PHP scripts for questionnaire sorting (https://sourceforge.net/projects/formsforsorting) and is currently writing a tool for preparing sorting data for analysis (http://www.sortkit.com/). Concerning Cluster Analysis and Multidimensional Scaling of dissimilarity matrices as well as factor analysis and correspondence analysis, a large number of software packages are available. NewMDSX (http://www.newmdsx.com/) contains programs for hierarchical and branch-and-bound clustering; for metric and non-metric scaling, for Simple Correspondence Analysis, for INDSCAL, and for comparison of configurations using Procrustean scaling (PINDIS). It also includes Takane's (1980, 1982) MD-/IDSORT program, designed for sorting data. SPSS and SAS, for example, also include hierarchical clustering, basic scaling (PROXSCAL), Correspondence Analysis and INDSCAL (Takane and Young, 1977) techniques. Concerning the determination of prototypes one can, for example, use the 'pam' object (Kaufman and Rousseeuw, 2005) of S and R (http://www.r-project.org/; R Development Core Team, 2006). It identifies medoids as a first step of clustering.
Most of the software packages for drag and drop sorting mentioned include analysis programs as well (like EZ-Calc of IBM), but these are usually restricted to a limited

number of analysis techniques. Syntagm (2005) has created software, which, prior to analysis, uses bar codes for data entry of paper card sorting. Its among the yet few programs to support hierarchies (like U-Sort and EZ-Calc of IBM) and multiple assignments, but in a currently (2005) limited way.
Fuzzy Cluster Analysis is part of some data mining tools and special software. Wagner and Wagner (2004) provide FCLUSTER (http://fuzzy.cs.uni-magdeburg.de/fcluster), a free software tool for fuzzy clustering. The 'fanny' object of R (http://www.r-project.org/), implementing the 'fanny' program of Kaufman and Rousseeuw (2005), is for Fuzzy Cluster Analysis as well, accepting two-way data as well as dissimilarity matrices for input. Harloff (2005b) has used it successfully for sorting data. PCKNOT is a program for Pathfinder Network Analysis, which may be obtained from Interlink (http://interlink.net/).

### 5.3 Hierarchical Cluster Analysis

Hierarchical agglomerative Cluster Analysis has been commonly applied to sorting data, for example in research on Human-Computer Interaction. But it has been used in situations as well where Fuzzy Cluster Analysis or Cover Set Cluster Analysis or Network Analysis was more appropriate, such as web page construction. Hierarchical Cluster Analysis is appropriate for data reflecting hierarchical relationships provided by hierarchy construction or weighted sorting.
As Harloff (2005a) pointed out, dendrograms resulting from Cluster Analysis may represent a structure too complex to be easily comprehensible for non-experts. Then, results need to be simplified to approach a final, usable structure. If, instead of a tree, an optimal partition or an optimal cover set shall be determined, variants of k-means cluster analysis like spectral cluster analysis may be used or interpretation restricted to one hierarchical level.

### 5.4 Fuzzy Cluster Analysis, Cover Set Cluster Analysis, Network Analysis

In terms of fuzzy sorting, items can be assigned to several groups simultaneously (item replication), or objects exhibit several attributes simultaneously, or attributes are assigned to several objects simultaneously etc. To analyse results connected to multiple assignments, two distinct models can be chosen. Like partitions, the fuzzy set model implies that any item is present just once in the whole item set, but items may be members of several groups simultaneously. Some portion of an item belongs to one group, while another portion belongs to another group. The sum of memberships of an item distributed over several groups is 100%. Such a relationship could for example be found in sorting data if participants don't make a definite assignment of some item "intermediate" between groups. This means that replicated cards symbolically represent a portion of an item only, in the simplest case the 1/m portion if m is the number of replications. The sum of memberships (portions) of each *item* equals that of all other items in the fuzzy set. Bezdek, Ehrlich and Full (1984) and recently Kaufman and Rousseeuw (2005) described Fuzzy Cluster Analysis algorithms.
For a cover set or network (graph) model, an item is fully placed in each of the groups it belongs to. Therefore, all *memberships* weigh equal, but different items receive different weights within the network according to the number of groups they are assigned to. Such a relationship could be found for example in web site information architecture, some items being placed in a permanently visible screen area

while other items are visible on single screens only. Network Analysis has been applied to sorting data (e.g., several contributions to Schvaneveldt, 1990: Pathfinder Network Analysis). Basically, Pathfinder Network Analysis reduces the complexity of relationships of objects, fitting the data to a set of theoretical restrictions. It displays a set of dominant relations as lines (edges) of a graph while intersections of lines are avoided. Another potentially suitable Network drawing tool is NETSCAL (Hutchinson, 1989), allowing for an input of asymmetric matrices. Hypertext links are directed and hence best represented by asymmetric matrices. - A couple of authors have combined displays of 2- or 3-dimensional Multidimensional Scaling results with network representations, improving the information content of their figures considerably (e.g. McDonald, Dearholt, Paap and Schvaneveldt, 1986; Hutchinson, 1989). Line width can be chosen proportional to the similarity of items, visualising the closeness of relationships, while display of lines can be restricted to similarities above a threshold (e.g. of cumulative similarity frequencies) to reduce complexity (e.g. Harloff, 2005b, 2005c). The area of item representations like circles can be chosen proportional to their weight (numbers and weights of connections to other items). Such figures are, however, a rather sketchy form of network analysis.

Variants of Cluster Analysis and Neural Networks can be adapted to produce cover sets, see Hartung and Elpelt (1999, pp 458-465), Ohlsson, Peterson and Söderberg (2001) and Rezek and Roberts (1997). Extended trees (Corter, 1996; Corter and Tversky, 1986) also allow for overlapping clusters, combined with hierarchies. However, to our knowledge a successful application to sorting data has not yet been done for any of these analysis methods. Capra (2005) used factor analysis of a similarity matrix for determination of cover sets within a hierarchy (see chapter 5.6).

## 5.5 Multidimensional Scaling (MDS)

Multidimensional Scaling was one of the first analysis methods applied to sorting data (see Coxon, 1982, 1999, for a more extended discussion). Nonmetric Multidimensional Scaling has the capability to deal with some missing data, an obvious advantage if participants omitted items erroneously or due to a lack of information.

There has been considerable discussion about some properties of binary data from co-occurrences in a partition as well as data from trees, and their consequences. The argument presented herein follows Bimler and Kirkland (2001, 2003). Obviously, items separated by a dissimilarity of 0 or by the smallest value of a rank order will not be discriminated by Nonmetric Multidimensional Scaling. Concerning similarity measures on trees, there is always a dominance of a group of large distance values between items. This can be easily seen in histograms of distance values showing a negatively skewed frequency or density distribution. After Nonmetric Multidimensional Scaling, a result is likely which shows dense clusters of items without internal structure and a circular (2 scaling dimensions) or spherical (3 scaling dimensions) arrangement of items in plots. Circles and spheres are the configurations that fit best to a negatively skewed frequency distribution of distance values. Their occurrence may be accepted as correct and unavoidable, as long as nominal or ordinal scales are clearly appropriate for structuring the domain. Otherwise, condensed groups and circles or spheres may be considered a methodological artefact. It can be avoided if a whole set of variants of sorting including opposite sorting is used (Bimler and Kirkland, 2001, 2003) or if a less skewed density distribution of distance values results from weighted sorting (Harloff, 2005a).

Concerning domains that are accepted to be represented well by circles (two dimensions), Arabie & Boorman (1973) support several earlier authors who had made use of histograms of polar coordinates instead of cartesian coordinates to detect clusters. Since circles have a constant radius, all information is contained in angular relationships. Concerning spheres (three dimensions) Bimler and Kirkland (2001, 2003) suggested to make use of a stereographic projection, e.g. into a Schmidt Net. A stereographic projection of a hemisphere perfectly represents area (Schmidt Net) or angular relationships (Wulff Net). Software packages, for example for geoscience purposes, are available for plotting into Schmidt Nets and contouring densities of projected directions. But the calculations for projection can be done on ones own using the Pascal code provided below. The calculation involves no loops and can be performed using a spreadsheet, too. Subsequently for example the 'kde2d' object of R (http://www.r-project.org/) can be used for density contouring. The Pascal procedure given calculates Xnew, Ynew coordinates in a Schmidt Net from Xold, Yold, Zold coordinates.

```
----------------------------------------------------------------------------------------------------
procedure stereo(Var Xnew, Ynew: real; Xold, Yold, Zold: real);
// Calculates Xnew, Ynew in a Schmidt Net from threedimensional coordinates Xold, Yold, Zold
// If the data set includes both positive and negative Zold values, you should assign different symbols
// to those points. Otherwise directions of the upper and lower hemispheres will be confused in inter-
// pretation.
const pi = 3.141592654; var r, p, l: real;
        begin
        if Xold <0 then l:=arctan(Yold/Xold)+pi
        else if Xold>0 then l:=arctan(Yold/Xold)
        else if (Xold=0) then
                begin
                if (Yold>0) then l:= pi/2
                else if (Yold<0) then l:= -pi/2
                else if (Yold=0) then l:= pi;
                        //artificial choice. Result not defined for this condition.
                end;
        if Zold<0 then p:= arctan(sqrt(sqr(Xold)+sqr(Yold))/Zold)+pi
        else if Zold>0 then p:= arctan(sqrt(sqr(Xold)+sqr(Yold))/Zold)
        else if Zold=0 then
                begin
                if (sqrt(sqr(Xold)+sqr(Yold))>0) then p:= pi/2
                else if (sqrt(sqr(Xold)+sqr(Yold))>0) then p:= -pi/2
                else if (sqrt(sqr(Xold)+sqr(Yold))=0) then p:= 0.0000000001;
                        //artificial choice. Result not defined for this condition.
                end;
        r := 2*sin(p/2);
        Xnew := r*cos(l); Ynew := r*sin(l);
        end; //procedure stereo. All angles given in radians.
----------------------------------------------------------------------------------------------------
```

Common Nonmetric Multidimensional Scaling is implemented in almost all current statistical software packages. A couple of special solutions have been published which overcome the problems described above for partition and tree data. Many of them are available through NewMDSX. Currently, no public software is available for the methods of Hojo (1993) and DeSarbo, Libby and Jedidi (1994).
If important, the participant structure is elicited together with the item structure. The Takane (1982) and DeSarbo et al. (1994) approaches are particularly remarkable

since they provide a three-way analysis of data. The DeSarbo et al. (1994) algorithm uses a probabilistic threshold approach. This makes it particularly useful for domains without clear-cut group boundaries, respecting possible individual differences.

## 5.6 Correspondence Analysis, Factor Analysis, Spectral Cluster Analysis

A two-way table of occurrences (Burt matrix) is often constructed as an intermediate step of Multiple Correspondence Analysis (MCA). Since a Burt matrix is an immediate result of sorting (visualised e.g. by a sorting questionnaire), MCA is a suitable analysis technique of sorting data.
Factor Analysis can be performed both on similarity matrices of items (Capra, 2005) if these matrices are invertible and positive semi definite, and on two-way tables of occurrences (Harloff 2006) with similar restrictions. Using Principal Component Analysis is an efficient means to extract partitions similar to those found by Multidimensional Scaling (Harloff, unpublished data). Spectral Cluster Analysis using an algorithmic description of Ng, Jordan and Weiss (2001) starting from a similarity matrix is efficient for retrieval of similar partitions, too (Harloff, unpublished data). Factor analysis offers a way to extract cover sets (Capra, 2005), which may be desirable in web site design. However, cover sets imply dependant factors of items and therefore factor analysis methods as well as factor rotation methods suited for oblique factors may be preferred (e.g. Harloff, 2006).

## 5.7 Participant structure (Q-Mode analysis)

Q-Mode analysis can be done as part of a two-way analysis of data (chapter 5.5) or using special similarity measures between sorting results of different participants. Coxon (1982, chapter 2.2.3, Table 2.6; also Coxon, 1999) has discussed this type of analysis and which measures should be used depending on the kind of data collected. Distance matrices of subjects (their trees, based on distance matrices of items) are produced and subsequently analysed using Cluster Analysis and Multidimenional Scaling. Still most important to Q-mode analysis are the papers of Arabie, Boorman and Olivier of 1973.
Arabie and Boorman (1973) presented a set of related measures suited to compare partition sortings. They are based on considerations about how many moves must be made at least to transform one partition into another. These measures are implemented in the SORTPAC package (Coxon). Hubert and Arabie (1986) gave an extended account on comparing partitions. Recently, Deibel, Anderson & Anderson (2005) seem to have reinvented the least-moves approach. - Boorman and Olivier (1973) presented a number of metric measures suited for trees and therefore on results of hierarchy construction or weighted sorting. Later, Olson and Biolsi (1991) covered the subject as well. Goldsmith and Davenport (1990) developed four (out of six) measures suited for networks from sorting data. Recently, Fossum and Haller (2005) added some very simple measures of similarity applicable to graphs based on partition sorting.
Other, simple similarity measures for ordinal and interval sorting data are Spearman's rho and Pearson's r correlation coefficients of paralleled similarity matrices, respectively. They can be easily computed using any statistics or mathematics software. Parallelized matrices can be used for hypothesis testing, too (Harloff 2005b; Fossum and Haller, 2005).

Raw (two-way) occurrence data can be used instead of co-occurrence data for Q-mode analysis as long as comparable groups exist for all partitions (closed sorting). Then, the same measures (Burton, 1975) are applicable like for R-mode analysis, but for the transposed matrix (Coxon, 1982). If two-way occurrence tables are factorised (chapter 5.6) scores and biplots can be used for Q-mode analysis of single clusters.
Like for R-Mode analysis, measures are usually taken for pair wise comparisons, not for triples or quadruples and so on.

## 5.8 Qualitative Analysis

Mathematical analysis does not by itself lead to an understanding of its results. It must be interpreted to become useful. Interpretation actually means qualitative reasoning.
While statistical analysis of averaged matrices is efficient in extracting factors, such statistical factors will rarely fit to a common language term. Common language labels and facets may be desirable, however, if the targeted categorisation shall be understandable and usable by a broad public. Then, qualitative separation of facets may rather lead to an appropriate solution. (Whether derived qualitatively or quantitatively, categorisations and labels intended for a broad public like those used in the Internet should be tested for usability prior to their release.)
Leaving the choice of the sorting variant and facet(s) to participants by instruction may be useful for pilot studies. But qualitative interview data will then be necessary to understand and to sort sorting data into groups of variants which subsequently can be analysed using one mathematical model. For example, rather fuzzy mathematical results are to be expected if all sorting results of figure 1 were analysed together using only one of the mentioned statistical processes.
Recently, a special issue of *Expert Systems* (22/3:2005) on card sorting included papers making use of some qualitative analysis techniques and providing examples.

## 6 References

A cared list of references may be found within the appendix to Coxon (1999) on the website http://www.methodofsorting.com/, which may be more complete than the information presented herein.


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# Appendix

## Document history

Version 1.0 of this document (How To Sort) was published by Joachim Harloff and Tony Coxon on November 1, 2005 on the website http://www.softuse.com using the GNU General Public License, version 1.2. It included an English and a German section. Copyright is held by Joachim Harloff and Tony Coxon. Version 1.0 is out of date now, but can be obtained from the authors on request.
Version 1.1 differs from version 1.0 in that the English version and the German translation are distributed in separate files, both available from http://www.methodofsorting.com. Chapter 2 has been rewritten. Some minor textual changes have been made throughout the text and updates made to recent literature as well as references to other sources available from http://www.methodofsorting.com/. Figure 1 as well as chapter 5.6 on Correspondence Analysis, Factor Analysis and Spectral Cluster Analysis have been added. Some subchapters and paragraphs within chapter 5 have been rearranged into a more logical order.

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